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An analysis on a supersymmetric chiral gauge theory with no flat direction

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Abstract

The low energy effective theory of the $N = 1$ supersymmetric $SU(5)$ gauge theory with chiral superfields in the 5^* and 10 representations is constructed. Instead of postulating the confinement of $SU(5)$ (confining picture), only the confinement of its subgroup $SU(4)$ is postulated (Higgs picture), and the effective fields are $SU(4)$-singlet but $SU(5)$-variant. The classical scalar potential which ensures unique supersymmetric vacuum at the classical level is incorporated into the Kähler potential of the effective fields. We show that supersymmetry and all other global symmetry are spontaneously broken. The scales of these symmetry breaking and the particle spectrum including Nambu-Goldstone particles are explicitly calculated, and no large scale hierarchy is found.

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I. INTRODUCTION

It is expected that there are some unknown dynamics of the gauge theory by which the problems in the standard model are solved. For instance, the natural scale hierarchy is expected in some class of chiral gauge theories ("tumbling gauge theories") [1], and the mass hierarchy of quarks and leptons may be explained by virtue of such dynamics. If the low-energy supersymmetry which may solve the naturalness problem exists, it may be spontaneously broken by the dynamics of the gauge theory. But, since the non-perturbative effect of the gauge theory is hard to evaluate, our efforts for the concrete model building have been limited.

The method proposed by Seiberg et al. [2–4] is remarkable, because it has a strong power to exactly determine the superpotentials of $N = 1$ supersymmetric gauge theories. Application of the method to various gauge theories will give us further new knowledge in addition to which has already been discovered on the supersymmetric QCD and so on [2–7]. Especially, the gauge theories with no flat direction in the scalar potential should be extensively studied to understand real physics, since they have unique vacuum where the supersymmetry may be spontaneously broken.

The $N = 1$ supersymmetric $SU(5)$ gauge theory with chiral superfields in the 5^* and 10 representations has been extensively studied. It is well known that the classical scalar potential of this theory has no flat direction, and ensures unique ground state with unbroken gauge symmetry. The spontaneous breaking of supersymmetry is suggested by the explicit instanton calculation [8,9] and the argument of the low energy effective theory [10]. In the effective theory of ref. [10] two effective fields, $S \sim \mathrm{tr}(W^2)$ and $\Phi_A \sim \Phi^T \Omega W^2 \Phi$ are introduced postulating the confinement of $SU(5)$, where W, Φ and Ω denote the chiral superfields of the $SU(5)$ gauge field strength, matters in the 5^* and 10 representations, respectively. It is argued that the supersymmetry is spontaneously broken by assuming a special Kähler potential and the general effective superpotential as far as the symmetry of the theory allows. In this picture all information on the dynamics is contained in the Kähler

potential which can not be determined by the argument of the symmetry.

In this paper we construct an effective theory from the different starting point. We do not postulate the confinement of $SU(5)$, but its subgroup $SU(4)$. Namely, we take Higgs picture instead of confining picture (complementarity [11,12]) expecting the gauge symmetry breaking of $SU(5) \to SU(4)$. This pattern of gauge symmetry breaking is suggested in the non-supersymmetric version of this theory: $SU(5)$ gauge theory with chiral fermions in the 5^* and 10 representations [1]. In this non-supersymmetric theory the most attractive channel hypothesis suggests a fermion pair condensation of the channel $10 \times 10 \to 5^*$ which causes the breaking of $SU(5) \to SU(4)$. Below its condensation scale, $SU(4)$ gauge theory with chiral fermion in the 4, 4^* and 1 representations is considered as the effective theory, and the subsequent QCD-like condensation of the channel $4 \times 4^* \to 1$ is expected by the most attractive channel hypothesis. The confinement of unbroken $SU(4)$ and a hierarchy between these two condensation scales are expected. Although the expectation of the gauge symmetry breaking of $SU(5) \to SU(4)$ may not be true in the supersymmetric theory, we construct the effective theory postulating the confinement of $SU(4)$, and justify the expectation by the result. Namely, if the vacuum expectation values of the effective fields, which are not $SU(5)$-singlet, support the gauge symmetry breaking of $SU(5) \to SU(4)$, we can consistently accept this expectation.

In the next section we introduce three effective fields, two of which are $SU(5)$-variant. Since this theory has no flat direction, all the effective fields are the massive fields except for the Nambu-Goldstone particles. We assume that global symmetry, $U(1)_R$ (R-symmetry) and $U(1)_A$ (chiral symmetry), are completely broken, and do not apply the 't Hooft anomaly matching condition [13]. Is is consistently justified by the finial result.

In section III the superpotential is uniquely determined using the method by Seiberg et al. The non-trivial Kähler potential is introduced to incorporate the effect of the classical scalar potential, by which the diverge vacuum expectation values are forbidden. The Kähler potential is not uniquely determined by the symmetry, but we assume most simple form by which the quantum scalar potential coincides with the classical one in the limit of $\Lambda \to 0$,

where Λ is the dynamical scale of $SU(5)$ gauge theory.

In section IV the vacuum and particle spectrum are examined. We find unique vacuum with finite vacuum expectation value of effective fields, where both two $U(1)$ global symmetry and supersymmetry are spontaneously broken. We show that two Nambu-Goldstone bosons appear corresponding with the spontaneous breaking of these two global $U(1)$ symmetry, one Nambu-Goldstone fermion appears corresponding with the breaking of supersymmetry, and all the other fields are massive. All the assumption on the symmetry breaking are consistently justified by the result.

In the last section we discuss the non-trivial Kähler potential, and conclude this paper.

Before closing this section, we settle the notation. The metric we use is $(1, -1, -1, -1)$, and the σ-matrixes for the two component spinor are $(\sigma_\mu)_{\alpha\dot{\beta}} = (1, \tau^i)$ and $(\bar{\sigma}_\mu)_{\dot{\alpha}\beta} = (1, -\tau^i)$, where τ^i is the Pauli matrix. The convention on the contraction of the index of two component spinor is

$$\theta\theta = \theta^{\dot{\alpha}}\theta_{\dot{\alpha}}, \quad \bar{\theta}\bar{\theta} = \bar{\theta}^{\alpha}\bar{\theta}_{\alpha}, \tag{1}$$

with $\theta^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\theta_{\dot{\beta}}$ and $\bar{\theta}^{\alpha} = \epsilon^{\alpha\beta}\bar{\theta}_{\beta}$, where $\epsilon^{\dot{\alpha}\dot{\beta}} = \epsilon_{\dot{\alpha}\dot{\beta}}$ and $\epsilon^{\alpha\beta} = \epsilon_{\alpha\beta}$. The integration over the spinors is defined as

$$\int d^2\theta\ \theta^2 = 1, \quad \int d^2\bar{\theta}\ \bar{\theta}^2 = 1. \tag{2}$$

The correspondence between the standard notation by Wess and Bagger [14] and ours is given in appendix A.

II. EFFECTIVE FIELDS

We first summarize the classical properties of the supersymmetric $SU(5)$ gauge theory with chiral superfields in the 5^* and 10 representations. The Lagrangian is written down as

$$\begin{aligned}\mathcal{L} = & \frac{1}{8g^2}\left\{\int d^2\theta\ \mathrm{tr}\left(W^{\dot{\alpha}}W_{\dot{\alpha}}\right) + \int d^2\bar{\theta}\ \mathrm{tr}\left(\bar{W}^{\alpha}\bar{W}_{\alpha}\right)\right\} \\ & - \int d^2\theta d^2\bar{\theta}\left\{\Phi^\dagger e^{2V^T}\Phi + \mathrm{tr}\left(\Omega^\dagger e^{-2V}\Omega e^{-2V^T}\right)\right\},\end{aligned} \tag{3}$$

where g denotes the gauge coupling constant, $W^{\dot{\alpha}}$ and $\bar{W}^{\alpha}$ are the chiral superfields of the $SU(5)$ gauge field strength composed by the vector superfield V, and Φ and Ω denote the chiral superfields of matters in the 5^* and 10 representations, respectively. At the classical level this theory has three global $U(1)$ symmetry

$$U(1)_X : \begin{cases} W_{\dot{\beta}}(y,\theta) \to e^{-i\alpha} W_{\dot{\beta}}(y,\theta e^{i\alpha}), \\ \Phi(y,\theta) \to \Phi(y,\theta e^{i\alpha}), \\ \Omega(y,\theta) \to \Omega(y,\theta e^{i\alpha}), \end{cases} \tag{4}$$

$$U(1)_{5^*} : \Phi(y,\theta) \to e^{i\beta}\Phi(y,\theta), \tag{5}$$

$$U(1)_{10} : \Omega(y,\theta) \to e^{i\gamma}\Omega(y,\theta), \tag{6}$$

but only two combinations of $Q_R \equiv Q_X + 9Q_{5^*} - Q_{10}$ and $Q_A \equiv 3Q_{5^*} - Q_{10}$ are anomaly free, where Q's denote the charge of $U(1)$ rotations. [1] The $U(1)$ symmetry defined by the charges Q_R and Q_A are called $U(1)_R$ and $U(1)_A$. The charges of the fields $W^{\dot{\alpha}}$, Φ and Ω are as follows.

	Q_R	Q_A
$W^{\dot{\alpha}}$	-1	0
Φ	9	3
Ω	-1	-1

(7)

The classical scaler potential comes from the D component of the vector superfields,

$$V_D = \frac{1}{2g^2} D^a D^a \tag{8}$$

with

$$\begin{aligned} D^a &= g^2 \left\{ A_\Phi^\dagger \left(-T^a\right)^T A_\Phi + \mathrm{tr}\left(A_\Omega^\dagger T^a A_\Omega\right) + \mathrm{tr}\left(A_\Omega^\dagger A_\Omega (T^a)^T\right) \right\} \\ &= g^2 \left\{ A_\Phi^\dagger T_{5^*}^a A_\Phi + A_\Omega^\dagger T_{10}^a A_\Omega \right\}, \end{aligned} \tag{9}$$

[1]The Θ-term is unphysical in this theory, because there is $U(1)$ symmetry which is explicitly broken only by the gauge anomaly.

where A_Φ, A_Ω are the scalar components of the chiral superfields Φ and Ω, and T^a, $T^a_{5^*}$ and T^a_{10} denote the generators of $SU(5)$ for the 5, 5^* and 10 representations, respectively. It is well known that $A_\Phi = 0$ and $A_\Omega = 0$ is the unique solution of the stationary condition of this potential, and classical vacuum is supersymmetric.

It is remarkable that no gauge invariant superpotential can be written down. Since all the gauge invariant holomorphic polynomial composed by the chiral superfields Φ and Ω vanish, we can not consider non-trivial superpotential even the non-renormalizable one. This fact seems to mean that the generation of the superpotential by the quantum effect can not be expected. On the other hand, the explicit non-perturbative calculation suggests that the supersymmetry is spontaneously broken [8,9]. Therefore, it is likely to imagine that the superfields Φ and Ω are not the good physical fields below the scale Λ (dynamical scale of $SU(5)$ gauge theory), and the low energy effective theory with different fields (composite fields) should be considered. The superpotential in the effective theory will lift up the vacuum, and supersymmetry is spontaneously broken. [2]

Now we consider what effective fields are appropriate in this theory. Since we do not know the symmetry of the exact vacuum unlike in the case of supersymmetric QCD, we must assume it. Namely, if we assume both or one of $U(1)_R$ or $U(1)_A$ symmetry is the symmetry of the vacuum, the effective fields must satisfy 't Hooft anomaly matching condition [13]. It is shown in ref. [10] that if both $U(1)_R$ and $U(1)_A$ symmetry is assumed as the symmetry of the vacuum, many effective fields with complicated quantum numbers have to be introduced. Such effective fields describe the particles which couple with the complicated high dimensional composite operators in the original theory. This situation is implausible, because the dynamics will make light composite particles which couple with low dimensional operators. In this paper we assume that both $U(1)_R$ and $U(1)_A$ symmetry are

[2]Of course, we can also imagine that the complicated Kähler potential triggers the spontaneous breaking of supersymmetry in both the original and effective theories [10].

spontaneously broken, and there is no massless fermions except for the Nambu-Goldstone fermion due to the spontaneous breaking of supersymmetry. Therefore, 't Hooft anomaly matching condition is not imposed. This assumption must be justified by the result of the analysis.

As explained in the previous section, the confinement of $SU(4)$, a subgroup of $SU(5)$, is assumed rather than the confinement of $SU(5)$ itself. This is also the assumption which must be justified by the result of the analysis. The guiding principle to introduce the $SU(5)$ variant effective fields is as follows.

We introduce only the effective fields which couple with the Lorentz invariant bi-linear operators composed by the three fields Φ, Ω and W in the original theory. In addition, we assume that the effective fields are in the smallest representations of $SU(5)$ in each bi-linear combinations. Namely, we consider the following effective fields.

$$\begin{aligned} X^i &\sim \epsilon^{ijklm}\Omega_{jk}\Omega_{lm} && 10 \times 10 \to 5^* \\ Y_i &\sim \Phi^j \Omega_{ji} && 5^* \times 10 \to 5 \\ S &\sim \mathrm{tr}\left(W^{\dot\alpha} W_{\dot\alpha}\right) && 24 \times 24 \to 1 \end{aligned} \tag{10}$$

The operator corresponding $5^* \times 5^* \to 10^*$ vanishes, since the superfields commute each other. Furthermore, since we assume the confinement of $SU(4)$, only the $SU(4)$-singlet parts of each effective fields are introduced as the effective fields. We introduce three fields

$$X \equiv X^{i=5}, \quad Y \equiv Y_{i=5}, \quad S, \tag{11}$$

as the effective fields below the scale Λ.

This guiding principle is supported by the following arguments. The classical scalar potential eq.(8) can be written like

$$\begin{aligned} V_D &= \frac{g^2}{2}\left[\left(A_\Omega^\dagger T_{10}^a A_\Omega\right)\left(A_\Omega^\dagger T_{10}^a A_\Omega\right) + \cdots\right] \\ &= \frac{g^2}{2}\left[-\lambda(10,10,5^*)\frac{1}{4!}(\epsilon\ A_\Omega A_\Omega)^i(\epsilon\ A_\Omega^\dagger A_\Omega^\dagger)_i - \lambda(10,10,50)\,|(A_\Omega A_\Omega)_{50}|^2 + \cdots\right], \end{aligned} \tag{12}$$

where

$$\lambda(10,10,5^*) = \frac{1}{2}\left\{C_2(10) + C_2(10) - C_2(5^*)\right\} = \frac{12}{5}, \tag{13}$$
$$\lambda(10,10,50) = \frac{1}{2}\left\{C_2(10) + C_2(10) - C_2(50)\right\} = -\frac{3}{5}, \tag{14}$$

and $C_2(r)$ denotes the coefficient of the second Casimir invariant of the representation r of $SU(5)$. [3] The method of the auxiliary field can be used to introduce the effective fields.

$$\begin{aligned}
V_D \to\ & -\frac{g^2}{2}\lambda(10,10,5^*)\frac{1}{4!}(\epsilon\ A_\Omega A_\Omega)^i(\epsilon\ A_\Omega^\dagger A_\Omega^\dagger)_i \\
& +\frac{1}{2}\lambda(10,10,5^*)\frac{1}{4!}\left\{g(\epsilon\ A_\Omega A_\Omega)^i - A_X^i\right\}\left\{g(\epsilon\ A_\Omega^\dagger A_\Omega^\dagger)_i - A_{Xi}^\dagger\right\} \\
& +\cdots \\
=\ & \frac{1}{2}\lambda(10,10,5^*)\frac{1}{4!}A_X^i A_{Xi}^\dagger \\
& -\frac{g}{2}\lambda(10,10,5^*)\frac{1}{4!}\left\{(\epsilon\ A_\Omega A_\Omega)^i A_{Xi}^\dagger + A_X^i(\epsilon\ A_\Omega^\dagger A_\Omega^\dagger)_i\right\} \\
& +\cdots,
\end{aligned} \tag{15}$$

where A_X^i denotes the scalar component of the effective field X^i. This result shows that if the coefficient λ is positive, the classical squared mass of the effective field becomes positive, and it is worth considering. The effective field in the 50 representation can not be considered, since $\lambda(10,10,50) < 0$, and its classical squared mass is negative. The same arguments are true for the effective fields composed by Φ and Ω. The effective field Y_i is worth considering, since $\lambda(5,10,5) = \frac{9}{5} > 0$, but the effective field in the 45 representation can not be considered, since $\lambda(5,10,45) = -\frac{1}{5} < 0$.

From this argument we obtain the classical scalar potential written by the effective fields A_X and A_Y.

$$V_{\text{classical}} = \lambda_X A_{Xi}^\dagger A_X^i + \lambda_Y A_Y^{\dagger i} A_{Yi} \to \lambda_X |A_X|^2 + \lambda_Y |A_Y|^2, \tag{16}$$

where $\lambda_X \equiv \frac{1}{2}\frac{1}{4!}\lambda(10,10,5^*)$ and $\lambda_Y \equiv \frac{1}{5}\lambda(5,10,5)$. The normalization of the effective fields X and Y is determined in this arguments.

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[3]The operator correspond to the channel $10 \times 10 \to 45$ vanishes because of the Bose statistics of the superfield.

$$X \equiv g\ \epsilon^{5jklm}\Omega_{jk}\Omega_{lm}. \quad Y \equiv g\ \Phi^j \Omega_{j5} \tag{17}$$

The normalization of the effective field S is determined in the next section.

III. SUPERPOTENTIAL AND KÄHLER POTENTIAL

Three effective chiral superfields X, Y and S with charges

$$\begin{array}{ccc} & Q_R & Q_A \\ X & -2 & -2 \\ Y & 8 & 2 \\ S & -2 & 0 \end{array} \tag{18}$$

are introduced in the previous section. We construct the effective theory using these fields which is effective below the scale Λ. Since the fields X and Y are not covariant under the $SU(5)$ transformation, the $SU(5)$ invariant Lagrangian can not be constructed. Imagine that the theory in which only the heavy $SU(5)/SU(4)$ gauge bosons ($SU(5)/SU(4)$ gauge bosons and would-be Nambu-Goldstone bosons) are integrated out, but Higgs is not integrated out. The theory we want to construct is neither the "linear σ-model" nor "non-linear σ-model" for $SU(5)$ gauge symmetry breaking, but "linear σ-model" for the breaking of the global symmetry $U(1)_R$, $U(1)_A$ and supersymmetry. Therefore, the invariance under all anomaly-free global symmetry is postulated.

The superpotential is uniquely determined using the method by Seiberg et al. [2–4]. The product XYS^3 is the unique independent holomorphic product which is invariant under $U(1)_R$ and $U(1)_A$ transformation, and S is the unique independent holomorphic quantity which is invariant under $U(1)_A$ transformation with $U(1)_R$-charge -2. [4] Therefore, the general form of the superpotential is

$$W = Sf\left(\frac{\Lambda^{13}}{XYS^3}\right) \tag{19}$$

[4] The superpotential must have $U(1)_R$-charge -2 in our notation

with a general holomorphic function f. Note that the power of Λ, 13, which comes from the dimensional analysis, is just the coefficient of the 1-loop β-function of the $SU(5)$ gauge coupling. In the weak coupling limit, $\Lambda \to 0$, this superpotential must coincide with the gauge kinetic term in the perturbatively-calculated Wilsonian action of the original theory

$$\mathcal{L}_{\text{gauge}}\Big|_{\Lambda\to 0} = -\frac{1}{64\pi^2} \ln \Lambda^{13} \ \mathrm{tr}\left(W^{\dot{\alpha}} W_{\dot{\alpha}}\right) + \text{h.c.}. \tag{20}$$

Therefore,

$$W = -\frac{1}{64\pi^2}\ S\ \ln\left(\frac{\Lambda^{13}}{XYS^3}\right) + S\tilde{f}\left(\frac{\Lambda^{13}}{XYS^3}\right), \tag{21}$$

where $\tilde{f}$ is a holomorphic function with $\lim_{z\to 0}\tilde{f}(z) = 0$. Here we take the normalization $S \equiv \mathrm{tr}(W^{\dot{\alpha}} W_{\dot{\alpha}})$. Moreover, if we assume that the massless degrees of freedom are only the Nambu-Goldstone particles, and all of them are described by the effective fields already introduced, the function $\tilde{f}$ should not have the singularities, and it is a constant. The constant is absorbed to the redefinition of Λ. Thus, we obtain

$$W = -\frac{1}{64\pi^2}\ S\ \ln\left(\frac{\Lambda^{13}}{XYS^3}\right). \tag{22}$$

This is the unique superpotential within our postulations.

We can obtain a scalar potential from the superpotential of eq.(22) assuming a naive Kähler potential

$$K_{\text{naive}} = \frac{1}{\Lambda^2}X^\dagger X + \frac{1}{\Lambda^2}Y^\dagger Y + \frac{1}{\Lambda^4}S^\dagger S, \tag{23}$$

where the normalization comes from that the effective fields X and Y, and S have dimension 2 and 3, respectively. But the solution of the stationary condition of the scalar potential is $\langle A_S\rangle \to 0$ and $\langle A_X\rangle, \langle A_Y\rangle \to \infty$ with $\ln(\Lambda^{13}/\langle A_X\rangle\langle A_Y\rangle\langle A_S\rangle^3) = 3$ (supersymmetric vacuum). This solution is not acceptable, because $\langle A_X\rangle$ and $\langle A_Y\rangle$ should not become infinity by virtue of the classical potential of eq.(16). The effect of the classical potential must be included. It is not the supersymmetric treatment to simply add the classical potential to the quantum potential, because the classical potential is the explicit soft breaking term of

supersymmetry. It is also impossible to include the classical effect as a constraint in the superpotential using the Lagrange multiplier like in the supersymmetric QCD with $N_f = N_c$, because this theory has no flat direction in the scalar potential. The remaining possibility is to consider the non-trivial Kähler potential.

The non-trivial Kähler potential of the form

$$K(X^\dagger X, Y^\dagger Y, S^\dagger S) = K_X(X^\dagger X) + K_Y(Y^\dagger Y) + K_S(S^\dagger S) \tag{24}$$

modifies the equation of motion of the auxiliary fields of each effective fields as

$$F_X^\dagger = -\left[\frac{\partial W}{\partial X}\right] \Big/ \left[\frac{\partial}{\partial(X^\dagger X)}\left((X^\dagger X)\frac{\partial K_X}{\partial(X^\dagger X)}\right)\right], \tag{25}$$

and so on, where [ ] denotes to take the scalar component. The scalar potential is given by

$$V = -\left[\frac{\partial W}{\partial X}\right]^\dagger F_X^\dagger - \left[\frac{\partial W}{\partial Y}\right]^\dagger F_Y^\dagger - \left[\frac{\partial W}{\partial S}\right]^\dagger F_S^\dagger. \tag{26}$$

We consider the Kähler potential

$$K_X(X^\dagger X) = \frac{1}{\Lambda^2} f(X^\dagger X)_{C_X/\Lambda}, \tag{27}$$

$$K_Y(Y^\dagger Y) = \frac{1}{\Lambda^2} f(Y^\dagger Y)_{C_Y/\Lambda}, \tag{28}$$

$$K_S(S^\dagger S) = \frac{1}{\Lambda^4} S^\dagger S, \tag{29}$$

with two real parameters C_X and C_Y, where a function $f(z)_a$ is defined by

$$f(z)_a \equiv \sum_{n=0}^{\infty}(-1)^n \frac{a^{2n} z^{2n+1}}{(2n+1)^2} = z\ F\left(1, \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -a^2 z^2\right), \tag{30}$$

and it satisfies

$$\frac{d}{dz}\left(z\ \frac{df(z)_a}{dz}\right) = \frac{1}{1+a^2 z^2}. \tag{31}$$

The function F is the generalized hypergeometric function. Note that the naive Kähler potential is contained in K_X and K_Y as the first term of the expansion.

The scalar potential is obtained as

$$V = \frac{\Lambda^4}{(64\pi^2)^2} \left| \ln \left(\frac{A_X A_Y A_S^3}{\Lambda^{13}} \right) + 3 \right|^2 + \frac{\Lambda^2}{(64\pi^2)^2} \left(\frac{|A_S|^2}{|A_X|^2} + \frac{|A_S|^2}{|A_Y|^2} \right)$$
$$+ \frac{C_X^2}{(64\pi^2)^2} \frac{|A_S|^2}{\Lambda^6} |A_X|^2 + \frac{C_Y^2}{(64\pi^2)^2} \frac{|A_S|^2}{\Lambda^6} |A_Y|^2. \tag{32}$$

The last two terms are the contribution of the non-trivial Kähler potential.

The two parameters C_X and C_Y are determined so that the potential of eq.(32) coincides with the classical one, eq.(16), in $\Lambda \to 0$ limit. The first two terms of the potential simply vanish in this limit, but the last two terms seem to be singular. As taking the limit, the vacuum expectation value of the effective field takes the place of its dynamical degree of freedom, and the effective field decouples. We assume that the effective field S firstly decouples because of its largest vacuum expectation value, though this should be justified by the result. The vacuum expectation value of S is proportional to Λ^3, and the coefficient r is independent of Λ, but it depends on C_X and C_Y. Therefore, we can determine these two parameters by the condition

$$\frac{C_X^2}{(64\pi^2)^2}\, r(C_X, C_Y)^2 = \lambda_X, \quad \frac{C_Y^2}{(64\pi^2)^2}\, r(C_X, C_Y)^2 = \lambda_Y. \tag{33}$$

In practice, we replace the parameters C_X and C_Y by λ_X, λ_Y and r, and iteratively solve the stationary condition of the scalar potential changing the value of r until finding the solution $\langle A_S \rangle = r\Lambda^3$.

IV. VACUUM AND MASS SPECTRUM

Now we solve the stationary condition of the potential of eq.(32). By using the phase rotation of $U(1)_R$ and $U(1)_A$, the vacuum expectation values of A_X and A_Y can be taken as real positive. The vacuum expectation value of A_S can have the imaginary part, but it is dynamically set to zero. Substituting $A_S = |A_S| e^{i\theta_S/\Lambda}$ into the potential, we obtain the potential for θ_S as

$$V_{\theta_S} = \frac{9\Lambda^2}{(64\pi^2)^2} \theta_S^2. \tag{34}$$

Therefore, $\langle\theta_S\rangle = 0$, and the vacuum expectation value of A_S also can be taken as real positive.

The stationary conditions on the three real positive valuables A_X, A_Y and A_S are

$$\frac{\Lambda^4}{(64\pi^2)^2}\left(\ln\frac{A_X A_Y A_S^3}{\Lambda^{13}}+3\right)-\frac{\Lambda^2}{(64\pi^2)^2}\frac{A_S^2}{A_X^2}+\frac{\lambda_X}{r^2}\frac{A_S^2}{\Lambda^6}A_X^2=0, \tag{35}$$

$$\frac{\Lambda^4}{(64\pi^2)^2}\left(\ln\frac{A_X A_Y A_S^3}{\Lambda^{13}}+3\right)-\frac{\Lambda^2}{(64\pi^2)^2}\frac{A_S^2}{A_Y^2}+\frac{\lambda_Y}{r^2}\frac{A_S^2}{\Lambda^6}A_Y^2=0, \tag{36}$$

$$\frac{3\Lambda^4}{(64\pi^2)^2}\left(\ln\frac{A_X A_Y A_S^3}{\Lambda^{13}}+3\right)+\frac{\Lambda^2}{(64\pi^2)^2}\left(\frac{A_S^2}{A_X^2}+\frac{A_S^2}{A_Y^2}\right)+\frac{\lambda_X}{r^2}\frac{A_S^2}{\Lambda^6}A_X^2+\frac{\lambda_X}{r^2}\frac{A_S^2}{\Lambda^6}A_Y^2=0, \tag{37}$$

respectively, where two parameters C_X and C_Y are replaced by the two known parameters λ_X and λ_Y, and an unknown parameter r.

Although it is difficult to get the complete analytical solution of these conditions, an analytical relation

$$\langle A_Y\rangle^2 = \frac{2\langle A_X\rangle^2}{(64\pi^2)^2(\lambda_X/r^2)\langle A_X\rangle^4/\Lambda^8 - 3} \tag{38}$$

is obtained. We substitute this relation into eqs.(35) and (36), and numerically solve them. We find the solution

$$\langle A_X\rangle \simeq (0.17)^2, \quad \langle A_Y\rangle \simeq (0.11)^2, \quad \langle A_S\rangle \simeq (0.31)^3, \tag{39}$$

in unit of Λ with $r = 0.03$. Note that consistently $r = 0.03 \simeq \langle A_S\rangle \simeq 0.031$, and the vacuum expectation value of the effective field A_S is the largest one, namely, $\langle A_S\rangle^{1/3} > \langle A_X\rangle^{1/2} > \langle A_Y\rangle^{1/2}$.

This solution is consistent with the assumption of breaking $SU(5) \to SU(4)$, since the effective field X, which is a component of the $SU(5)$-variant effective field in the 5^* representation, obtains the vacuum expectation value. It can be considered that the vacuum expectation value of the effective field Y is caused by the dynamics of the effective $SU(4)$ gauge theory below the scale of the breaking of $SU(5)$ triggered by $\langle X\rangle \neq 0$. The assumption on the breaking of the global $U(1)_R \times U(1)_A$ symmetry is also confirmed. Since

the vacuum expectation value of the effective filed S means the gaugino pair condensation, the spontaneous breaking of supersymmetry is expected through Konishi anomaly [15]. In fact, the vacuum energy density is not zero, $V_{\text{vacuum}} \simeq (0.16)^4$ in unit of Λ. The vacuum energy density is the order parameter of supersymmetry breaking with absolute normalization. Taking $\langle A_X \rangle$ as the order parameter of gauge symmetry breaking, we find that both supersymmetry and gauge symmetry are spontaneously broken at almost the same scale.

The mass spectrum of the effective fields can be explicitly calculated. On boson fields, it is convenient to consider the non-linear realization of the global $U(1)_R \times U(1)_A$ symmetry

$$A_X = \Lambda \ \phi_X e^{i\theta_X/\Lambda}, \quad A_Y = \Lambda \ \phi_Y e^{i\theta_Y/\Lambda}, \quad A_S = \Lambda^2 \ \phi_S e^{i\theta_S/\Lambda}, \tag{40}$$

where $\phi_{X,Y,S}$ and $\theta_{X,Y,S}$ are the real scalar fields with dimension one. By substituting this expression to the scaler potential of eq.(32), we obtain

$$\begin{aligned} V = & \frac{\Lambda^4}{(64\pi^2)^2} \left\{ \left(\ln \frac{\phi_X \phi_Y \phi_S^3}{\Lambda^5} + 3 \right)^2 + \frac{1}{\Lambda^2} \left(\theta_X + \theta_Y + 3\theta_S \right)^2 \right\} \\ & + \frac{\Lambda^4}{(64\pi^2)^2} \left(\frac{\phi_S^2}{\phi_X^2} + \frac{\phi_S^2}{\phi_Y^2} \right) + \frac{\lambda_X}{r^2} \phi_S^2 \phi_X^2 + \frac{\lambda_Y}{r^2} \phi_S^2 \phi_Y^2 . \end{aligned} \tag{41}$$

The mass matrix for the fields $\theta_{X,Y,S}$ is given by

$$\mathcal{L}^{\theta}_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \theta_X & \theta_Y & \theta_S \end{pmatrix} M_\theta^2 \begin{pmatrix} \theta_X \\ \theta_Y \\ \theta_S \end{pmatrix}, \quad M_\theta^2 = \frac{2\Lambda^2}{(64\pi^2)^2} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix}. \tag{42}$$

Two of three eigenvalues are zero which are corresponding to the Nambu-Goldstone bosons of $U(1)_R$ and $U(1)_A$ breaking, and remaining eigenvalue is $m_\theta^2 = 22\Lambda^2/(64\pi^2)^2 \simeq (0.0074\Lambda)^2$. The smallness of this value can be understood by considering that it is corresponding to the mass of the pseudo-Nambu-Goldstone boson due to the anomalous global $U(1)$ symmetry breaking.

The mass matrix for the fields $\phi_{X,Y,S}$ can be obtained by differentiating the potential of eq.(41).

$$
\mathcal{L}^{A}_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \phi_X & \phi_Y & \phi_S \end{pmatrix} M_A^2 \begin{pmatrix} \phi_X \\ \phi_Y \\ \phi_S \end{pmatrix}, \tag{43}
$$

where M_A^2 is given by

$$
\frac{1}{(64\pi^2)^2}
\begin{pmatrix}
\begin{array}{l} 2\Big\{ \frac{1}{A_X^2} + \frac{3A_S^2}{A_X^4} \\ +(64\pi^2)^2 \frac{\lambda_X}{r^2} A_S^2 \\ - \frac{\ln(A_X A_Y A_S^3)+3}{A_X^2} \Big\} \end{array} & \frac{1}{A_X A_Y} & \begin{array}{l} \frac{3}{A_S A_X} - \frac{2A_S}{A_X^3} \\ +(64\pi^2)^2 \frac{2\lambda_X}{r^2} A_S A_X \end{array} \\
\frac{1}{A_X A_Y} & \begin{array}{l} 2\Big\{ \frac{1}{A_Y^2} + \frac{3A_S^2}{A_Y^4} \\ +(64\pi^2)^2 \frac{\lambda_Y}{r^2} A_S^2 \\ - \frac{\ln(A_X A_Y A_S^3)+3}{A_Y^2} \Big\} \end{array} & \begin{array}{l} \frac{3}{A_S A_Y} - \frac{2A_S}{A_Y^3} \\ +(64\pi^2)^2 \frac{2\lambda_Y}{r^2} A_S A_Y \end{array} \\
\begin{array}{l} \frac{3}{A_S A_X} - \frac{2A_S}{A_X^3} \\ +(64\pi^2)^2 \frac{2\lambda_X}{r^2} A_S A_X \end{array} & \begin{array}{l} \frac{3}{A_S A_Y} - \frac{2A_S}{A_Y^3} \\ +(64\pi^2)^2 \frac{2\lambda_Y}{r^2} A_S A_Y \end{array} & \begin{array}{l} 2\Big\{ \frac{1}{A_X^2} + \frac{1}{A_Y^2} \\ +(64\pi^2)^2 \left(\frac{\lambda_X}{r^2} A_X^2 + \frac{\lambda_Y}{r^2} A_Y^2 \right) \\ - \frac{3\ln(A_X A_Y A_S^3)}{A_S^2} \Big\} \end{array}
\end{pmatrix}. \tag{44}
$$

Here Λ is set to unity, and A_X, A_Y and A_S denote the vacuum expectation values of eq.(39). Though the analytic expression of the mass matrix is very complicated, its three eigenvalues can be numerically estimated as

$$
m_A^2 \simeq (0.45)^2, \quad (0.73)^2, \quad (1.5)^2, \tag{45}
$$

in unit of Λ. No large hierarchy is realized, but all are rather heavy.

There are two contributions to the masses of the fermion components, $\Lambda\psi_X$, $\Lambda\psi_Y$ and $\Lambda^2\psi_S$, of the effective chiral superfields, X, Y and S, respectively, where all ψ's have dimension 3/2. One comes from the superpotential, and another comes from the Kähler potential. The superpotential of eq.(22) gives the mass matrix

$$
\mathcal{L}^{W}_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \psi_X^{\dot\alpha} & \psi_Y^{\dot\alpha} & \psi_S^{\dot\alpha} \end{pmatrix} M_\psi^W \begin{pmatrix} \psi_{X\dot\alpha} \\ \psi_{Y\dot\alpha} \\ \psi_{S\dot\alpha} \end{pmatrix} + \text{h.c.}, \tag{46}
$$

$$M_\psi^W = \frac{1}{64\pi^2} \begin{pmatrix} \frac{3\Lambda^4}{\langle A_S \rangle} & \frac{\Lambda^3}{\langle A_X \rangle} & \frac{\Lambda^3}{\langle A_Y \rangle} \\ \frac{\Lambda^3}{\langle A_X \rangle} & -\frac{\Lambda^2 \langle A_S \rangle}{\langle A_X \rangle^2} & 0 \\ \frac{\Lambda^3}{\langle A_Y \rangle} & 0 & -\frac{\Lambda^2 \langle A_S \rangle}{\langle A_Y \rangle^2} \end{pmatrix}. \tag{47}$$

The mass matrix emerged from the Kähler potential of eq.(24) has diagonal from. Since the Kähler potential for S is naive one, there is no contribution to the mass of ψ_S. The contribution to the masses of remaining ψ_X and ψ_Y is

$$\begin{aligned} \mathcal{L}_{\text{mass}}^K = & \frac{\left\langle \frac{\partial^2 K_X}{\partial (X^\dagger X)^2} + \frac{1}{2}(X^\dagger X) \frac{\partial^3 K_X}{\partial (X^\dagger X)^3} \right\rangle}{\left\langle \frac{\partial}{\partial (X^\dagger X)} \left((X^\dagger X) \frac{\partial K_X}{\partial (X^\dagger X)} \right) \right\rangle} \left\langle \frac{\partial W}{\partial X} X^\dagger \right\rangle \psi_X^{\dot\alpha} \psi_{X\dot\alpha} + \text{h.c.} \\ & + \frac{\left\langle \frac{\partial^2 K_Y}{\partial (Y^\dagger Y)^2} + \frac{1}{2}(Y^\dagger Y) \frac{\partial^3 K_Y}{\partial (Y^\dagger Y)^3} \right\rangle}{\left\langle \frac{\partial}{\partial (Y^\dagger Y)} \left((Y^\dagger Y) \frac{\partial K_Y}{\partial (Y^\dagger Y)} \right) \right\rangle} \left\langle \frac{\partial W}{\partial Y} Y^\dagger \right\rangle \psi_Y^{\dot\alpha} \psi_{Y\dot\alpha} + \text{h.c.}, \end{aligned} \tag{48}$$

where $\langle\ \rangle$ denotes to take the vacuum expectation value of the scalar component. Since the function $f(z)_a$ satisfies the simple formula

$$\frac{d^2 f(z)_a}{dz^2} + \frac{1}{2} z \frac{d^3 f(z)_a}{dz^3} = -\frac{a^2 z}{(1+a^2 z^2)^2}, \tag{49}$$

we obtain

$$\mathcal{L}_{\text{mass}}^K = -\frac{1}{64\pi^2} \frac{C_X^2 \langle A_X \rangle^2 \langle A_S \rangle / \Lambda^6}{1 + C_X^2 \langle A_X \rangle^4 / \Lambda^8} \psi_X^{\dot\alpha} \psi_{X\dot\alpha} - \frac{1}{64\pi^2} \frac{C_Y^2 \langle A_Y \rangle^2 \langle A_S \rangle / \Lambda^6}{1 + C_Y^2 \langle A_Y \rangle^4 / \Lambda^8} \psi_Y^{\dot\alpha} \psi_{Y\dot\alpha} + \text{h.c.}. \tag{50}$$

Therefore, total mass matrix becomes

$$M_\psi = \frac{1}{64\pi^2} \begin{pmatrix} \frac{3\Lambda^4}{\langle A_S \rangle} & \frac{\Lambda^3}{\langle A_X \rangle} & \frac{\Lambda^3}{\langle A_Y \rangle} \\ \frac{\Lambda^3}{\langle A_X \rangle} & -\frac{\Lambda^2 \langle A_S \rangle}{\langle A_X \rangle^2} \frac{1 - C_X^2 \langle A_X \rangle^4/\Lambda^8}{1 + C_X^2 \langle A_X \rangle^4/\Lambda^8} & 0 \\ \frac{\Lambda^3}{\langle A_X \rangle} & 0 & -\frac{\Lambda^2 \langle A_S \rangle}{\langle A_Y \rangle^2} \frac{1 - C_Y^2 \langle A_Y \rangle^4/\Lambda^8}{1 + C_Y^2 \langle A_Y \rangle^4/\Lambda^8} \end{pmatrix}. \tag{51}$$

One can easily check that this mass matrix has one zero eigenvalue, which is corresponding to the Nambu-Goldstone fermion of supersymmetry breaking, by using the stationary conditions of eqs.(35), (36) and (37) with $C_{X,Y}^2 = (64\pi^2)^2 \lambda_{X,Y}/r^2$. The other two eigenvalues are numerically given by

$$m_\psi \simeq 0.33, \quad 0.091, \tag{52}$$

in unit of Λ. There is no large hierarchy.

V. CONCLUSION

An effective theory of the supersymmetric $SU(5)$ gauge theory with chiral superfields in the 5^* and 10 representations is constructed within two important postulations. One important postulation is on the symmetry breaking of the gauge and global symmetry. We postulate the spontaneous gauge symmetry breaking of $SU(5) \to SU(4)$ and confinement of $SU(4)$. It is also postulated that the global symmetry $U(1)_R \times U(1)_A$ is completely broken. Basing on this postulation, the effective fields which are not $SU(5)$-singlet but singlet under the transformation of its subgroup $SU(4)$ are introduced without imposing 't Hooft anomaly matching condition. The postulation on these symmetry breaking is consistently justified by the result.

Another important postulation is to introduce the non-trivial Kähler potential so that the quantum scalar potential coincides with the classical one in the limit of $\Lambda \to 0$. It is notable that the first term of the expansion of the introduced Kähler potential is the naive one which gives normal kinetic terms of the component fields.

The Kähler potential introduced in this paper may be the unique one which satisfies the conditions:

1. Coincide with the naive Kähler potential in the limit of weak field strength,

2. Scalar potential coincides with the classical one in the limit of weak coupling.

We can try to introduce the Kähler potential by which the classical scalar potential is trivially incorporated into the quantum scalar potential. Such Kähler potential must have the form

$$K(X^\dagger X, Y^\dagger Y, S^\dagger S) = K_{XS}(X^\dagger X, S^\dagger S) + K_{YS}(Y^\dagger Y, S^\dagger S), \tag{53}$$

and the equation of motion of the auxiliary fields of each effective fields become

$$F_X^\dagger = -\frac{\left[\frac{\partial W}{\partial X}\right]\left[\frac{\partial}{\partial (S^\dagger S)}\left((S^\dagger S)\frac{\partial K_{XS}}{\partial (S^\dagger S)}\right)\right] - \left[\frac{\partial^2 K_{XS}}{\partial (X^\dagger X)\partial (S^\dagger S)}\right]\left[\frac{\partial W}{\partial S}\right]\left[X^\dagger S\right]}{\left[\frac{\partial}{\partial (X^\dagger X)}\left((X^\dagger X)\frac{\partial K_{XS}}{\partial (X^\dagger X)}\right)\right]\left[\frac{\partial}{\partial (S^\dagger S)}\left((S^\dagger S)\frac{\partial K_{XS}}{\partial (S^\dagger S)}\right)\right] - \left[\frac{\partial^2 K_{XS}}{\partial (X^\dagger X)\partial (S^\dagger S)}\right]^2 \left[X^\dagger X S^\dagger S\right]}, \tag{54}$$

and so on. The potential given by eq.(26) becomes extremely complicated one, and some undesirable terms, which are singular in the limit of $\Lambda \to 0$ keeping the dynamical degrees of freedom of X and Y alive, will emerge.

The mass spectrum of the effective fields which describe composite particles are explicitly calculated. It is analytically shown that the three Nambu-Goldstone particles which is corresponding with the spontaneous breaking of supersymmetry and $U(1)_R \times U(1)_A$ symmetry appear in the spectrum. There is no large scale hierarchy in the mass spectrum, but we can see that bosons except for the (pseudo-)Nambu-Goldstone bosons are heavier than the fermions.

It is expected that the method developed in this paper is applied to the other (chiral) gauge theories with no flat direction, and some new dynamics are found, by which the problems of the standard model are solved.

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APPENDIX A: NOTATION

Followings are the correspondence between the notation by Wess and Bagger [14] and ours.

On the metric and spinors:

$$\eta^{mn}\Big|_{W-B} = -g^{\mu\nu}. \tag{A1}$$

$$\epsilon^{\alpha\beta}\Big|_{W-B} = \epsilon^{\alpha\beta}, \qquad \epsilon_{\alpha\beta}\Big|_{W-B} = -\epsilon_{\alpha\beta}. \tag{A2}$$

$$\left(\sigma^m\right)_{\alpha\dot\beta}\Big|_{W-B} = -\left(\sigma^\mu\right)_{\alpha\dot\beta}, \qquad \left(\bar\sigma^m\right)^{\dot\alpha\beta}\Big|_{W-B} = -\left(\bar\sigma^\mu\right)^{\dot\alpha\beta}. \tag{A3}$$

$$\theta^{\alpha}\Big|_{W-B} = \bar{\theta}^{\alpha}, \qquad \bar{\theta}^{\dot{\alpha}}\Big|_{W-B} = \theta^{\dot{\alpha}}. \tag{A4}$$

$$\theta\theta\Big|_{W-B} = \bar{\theta}\bar{\theta} = \bar{\theta}^{\alpha}\bar{\theta}_{\alpha}, \qquad \bar{\theta}\bar{\theta}\Big|_{W-B} = -\theta\theta = -\theta^{\dot{\alpha}}\theta_{\dot{\alpha}}. \tag{A5}$$

$$d^2\theta\Big|_{W-B} = d^2\bar{\theta}, \qquad d^2\bar{\theta}\Big|_{W-B} = -d^2\theta. \tag{A6}$$

On the chiral superfields:

$$W_{\alpha}(x,\theta)\Big|_{W-B} = \frac{1}{2}\bar{W}_{\alpha}(x,\bar{\theta}), \qquad \bar{W}_{\dot{\alpha}}(x,\bar{\theta})\Big|_{W-B} = \frac{1}{2}W_{\dot{\alpha}}(x,\theta). \tag{A7}$$

$$\Phi(y,\theta)\Big|_{W-B} = \Phi^{\dagger}(y^{\dagger},\bar{\theta}), \qquad \Phi^{\dagger}(y^{\dagger},\bar{\theta})\Big|_{W-B} = \Phi(y,\theta). \tag{A8}$$

$$y^{m}\Big|_{W-B} \equiv x^{m} + i\theta\sigma^{m}\bar{\theta}\Big|_{W-B} = y^{\dagger\mu} \equiv x^{\mu} - i\bar{\theta}\sigma^{\mu}\theta. \tag{A9}$$

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